Answers to Coursebook questions – Chapter 5.5

1 **a** The top resistors are in series and give a total of $8.0\ \Omega$. The bottom 2 similarly give a total of $4.0\ \Omega$. These are in parallel for a grand total of $\frac{1}{8}+\frac{1}{4}=\frac{3}{8} \Rightarrow R=2.7\ \Omega$.

b The $6.0\ \Omega$ and $4.0\ \Omega$ resistors are in parallel for a total of $\frac{1}{6}+\frac{1}{4}=\frac{5}{12} \Rightarrow R=2.4\ \Omega$.
The rest are in series for a grand total of $2.4+2.0+8.0=12.4\ \Omega$.

c All are in parallel for a total of $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1 \Rightarrow R=1.0\ \Omega$.

2 The 2 $20\ \Omega$ resistors and the $10\ \Omega$ resistor are in series for a total of $50\ \Omega$.
This is in parallel with the $30\ \Omega$ for a total of $\frac{1}{50}+\frac{1}{30}=\frac{8}{150} \Rightarrow R=18.75\ \Omega$.
This is now in series with the rest for a total of $18.75+20=38.75 \approx 39\ \Omega$.

3 Each of the 2 blocks consists of 3 resistors in parallel for a total of $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{1}{2} \Rightarrow R=2.0\ \Omega$. The 2 blocks are in series for a total of $4.0\ \Omega$.

4 The largest resistance is obtained when all are in series for a total of $100\ \Omega$.
The smallest is when all in parallel for a total of $100\times\frac{1}{1}=100 \Rightarrow R=0.01\ \Omega$.

5 Each half has resistance $\frac{R}{2}$. When joined in parallel the total is
$\frac{1}{R/2}+\frac{1}{R/2}=\frac{4}{R} \Rightarrow R_T=\frac{R}{4}$.

6 <u>Top circuit</u>: The total resistance of the 2 parallel resistors is $\frac{1}{10}+\frac{1}{20}=\frac{3}{20} \Rightarrow R=6.67\ \Omega$ for a grand total of $2.0+6.67=8.87\ \Omega$. The current leaving the battery is $I=\frac{12}{8.67}=1.38\text{ A}$. This is the current in the $2.0\ \Omega$ resistor. The potential difference across it is then $V=IR=1.38\times 2.0=2.8\text{ V}$. The current will split as it enters the 2 parallel resistors in the ratio $2:1$ with the $20\ \Omega$ resistor getting the lesser current. Since $\frac{1.38}{3}=0.46\text{ A}$, the $20\ \Omega$ resistor gets a current of 0.46 A and the $10\ \Omega$ resistor gets a current of $2\times 0.46=0.92\text{ A}$. Their common potential difference is $V=IR=0.46\times 20=(0.92\times 10=1.38\times 6.67)=9.2\text{ V}$.

Bottom circuit: The total resistance is: The 2 $4.0\ \Omega$ resistors are in parallel for a total of $\frac{1}{4.0}+\frac{1}{4.0}=\frac{1}{2.0}\Rightarrow R=2.0\ \Omega$ and this is in series with the rest for a total of $2.0+2.0+2.0=6.0\ \Omega$. The total current leaving the battery is then $I=\frac{6.0}{6.0}=1.0\ \text{A}$. This is the current in the 2 $2.0\ \Omega$ resistors, and the potential difference across each is then $V=IR=1.0\times 2.0=2.0\ \text{V}$. The potential difference across the parallel combination is then $V=6.0-2.0-2.0=2.0\ \text{V}$ and so the current in each of the $4.0\ \Omega$ resistors is $\frac{2.0}{4.0}=0.50\ \text{A}$.

7 The total resistance in the circuit will be $R+2$.

The current in the circuit is $I=\frac{10}{(R+2)}$ and so the power dissipated in the resistor *R* will be $P=R\left(\frac{10}{R+2}\right)^2$. When graphed this gives:

This shows that the power is maximum when the resistor *R* is equal to $2.0\ \Omega$, the value of the internal resistor.

Note: This is a general result. For an internal resistance r and emf V, the current is $I=\frac{V}{R+r}$ and so the power in the resistor *R* is $P=R\left(\frac{V}{R+2}\right)^2$.
To find the maximum power we must differentiate the power with respect to *R* (a simple exercise of what you have learned about calculus in your math classes):
$\frac{\mathrm{d}P}{\mathrm{d}R}=\frac{V^2}{(R+r)^2}-\frac{2RV^2}{(R+r)^3}=V^2\frac{R+r-2R}{(R+r)^3}=V^2\frac{r-R}{(R+r)^3}$. This is zero when $R=r$.
It can easily be checked that this corresponds to a maximum in the power.

8 It will be the same since the potential difference across each light bulb is always 9.0 V.

9 **a** From $P = VI$ we get $I = \frac{1200}{220} = 5.45\text{ A}$ for the toaster and

$I = \frac{500}{220} = 2.27\text{ A}$ for the mixer.

b The total power is 1700 W and so in 1 h the energy used is 1.7 kW h.

10 The total resistance is found as:

$40.0 + 20.0 = 60.0\ \Omega$

$\frac{1}{60.0} + \frac{1}{60.0} = \frac{1}{30.0} \Rightarrow R = 30.0\ \Omega$

$30.0 + 60.0 + 57.0 + 3.0 = 150\ \Omega$

Hence the total current is $I = \frac{12.0}{150} = 0.080\text{ A}$.
This is the current in all the series resistors.
In the parallel block the current splits equally into 0.040 A in the upper and lower branch.
The total power is equal to the emf times the total current, i.e.
$P_T = \varepsilon I = 12 \times 0.080 = 0.96\text{ W}$.

11 **a** From $P = VI$ we get $I = \frac{2000}{220} = 9.09 \approx 9.1\text{ A}$.

b $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = 24.2 \approx 24\ \Omega$.

c The energy needed to warm the water is $Q = mc\Delta\theta = 2.0 \times 4200 \times (90 - 15) = 6.3 \times 10^5\text{ J}$. Since the power is 2000 W the time required is given by $2000 \times t = 6.3 \times 10^5 \Rightarrow t = 315\text{ s} = 5.25\text{ min}$.

d The energy used is $2000\text{ W} \times 315\text{ s} = 2.0\text{ kW} \times \frac{315}{3600}\text{ h} = 0.175\text{ kW h}$ and so the cost is $0.175 \times \$0.10 = 0.0175 \approx \$0.02$.

12 **a** From $P=\frac{V^2}{R}$ we can find the resistance of each light bulb:

$R_1=\frac{V^2}{P}=\frac{220^2}{60}\approx 806.7\,\Omega$ and $R_2=\frac{V^2}{P}=\frac{220^2}{75}\approx 645.3\,\Omega$

So at the new voltage:

$I_1=\frac{110}{806.7}=0.136\approx 0.14\text{ A}$ and $I_2=\frac{110}{645.3}=0.170\approx 0.17\text{ A}$

b At 110 V the power dissipated in each light bulb would be less than the rated value of the power at 220 V. Since $P=\frac{V^2}{R}$ and the voltage is reduced by a factor of 2, the power will be reduced by a factor of $2^2=4$. So for equal times, the cost at 220 V is 4 times as large (but remember the power is 4 times less at 110 V).

13 Each appliance draws a current of

$I_1=\frac{60}{220}=0.27\text{ A}$, $I_2=\frac{500}{220}=2.27\text{ A}$ and $I_3=\frac{60}{220}=5.45\text{ A}$ for a total current leaving the plug of $0.27+2.27+5.45=7.99\approx 8.0\text{ A}<10\text{ A}$, so the fuse will not blow.

14 Each appliance draws a current of $I_1=\frac{1200}{220}=5.45\text{ A}$ and $I_2=\frac{1000}{220}=4.55\text{ A}$ for a total current leaving the plug of $5.45+4.55=10\text{ A}>9.0\text{ A}$, so the fuse will blow if both are used at the same time.

15 **a** Reading off the graph, $R=30\text{ k}\Omega$.

b The total resistance is $50\text{ k}\Omega$ and so the current is $I=\frac{10}{50\times 10^3}=2.0\times 10^{-4}\text{ A}$.

16 The total resistance of the circuit will decrease when the temperature increases and so the current in the circuit (and hence the lamp) will increase. Hence the brightness will go up.

17 **a** $P=VI=220\times 20=4.4\text{ kW}$.

b We need energy to warm the water to $100\,^\circ\text{C}$ and then vaporize the water. This thermal energy is $mc\Delta\theta+mL=2.0\times 4200\times 60+2.0\times 2257\times 10^3=5.02\times 10^6\text{ J}$. Thus the time taken will be $4400\times t=5.02\times 10^6\Rightarrow t=1140\text{ s}=19\text{ min}$.

18 We have that $12=I(R_1+R_2)$ and $\varepsilon=IR_2$, where R_1 is the resistance of wire AC and R_2 the resistance of wire BC. Thus $\frac{\varepsilon}{12}=\frac{R_2}{R_1+R_2}$. But the resistances are proportional to the lengths and so $\frac{\varepsilon}{12}=\frac{54}{100}\Rightarrow\varepsilon=6.48\text{ V}$.

19 The resistances of each light bulb can be found from $R_1 = \frac{V^2}{P} = \frac{220^2}{60} \approx 806.7\ \Omega$ and $R_2 = \frac{V^2}{P} = \frac{220^2}{75} \approx 645.3\ \Omega$. When in series the total resistance will be $806.7 + 645.3 = 1452\ \Omega$ and so the current in the circuit will be $I = \frac{220}{1452} = 0.1515\ \text{A}$. The power in each light bulb is then $P_1 = 806.7 \times 0.1515^2 \approx 18.5\ \text{W}$ and $P_2 = 645.3 \times 0.1515^2 = 14.8\ \text{W}$.

20 The current drawn by each light bulb is $I = \frac{75}{220} = 0.341\ \text{A}$ and so the number of light bulbs is $\frac{100}{0.341} = 293$ (this is a big house).

21 **a** The total resistance of the voltmeter and the resistance it is connected to is $\frac{1}{200} + \frac{1}{200} = \frac{1}{R} \Rightarrow R = 100\ \Omega$.

Hence the voltmeter would read $\frac{100}{200+100} \times 12.0 = 4.0\ \text{V}$.

b The total resistance of the circuit is $200 + 100 = 300\ \Omega$ and so the current read by the ammeter is $\frac{12.0}{300} = 0.040\ \text{A}$.

c The resistance of the circuit is now $400\ \Omega$. The voltmeter would read $\frac{200}{200+200} \times 12.0 = 6.0\ \text{V}$. The ammeter would read $\frac{12.0}{400} = 0.030\ \text{A}$.

22 The total resistance is found in stages as:

$$\frac{1}{80} + \frac{1}{120} = 0.02083 \Rightarrow R = 48\ \Omega$$

$$48 + 12 = 60\ \Omega$$

$$\frac{1}{60} + \frac{1}{40} = 0.04167 \Rightarrow R = 24\ \Omega$$

Hence the current is $I = \frac{120}{24} = 5.0\ \text{A}$.

23 **a** The current is $I = \frac{12.0}{8.0 + 0.50} = 1.412\ \text{A}$ and so the power in the load is $P = 8.0 \times 1.412^2 \approx 15.9\ \text{W}$.

b The power in the internal resistor is $P = 0.5 \times 1.412^2 \approx 0.997\ \text{W}$ and so in 10 minutes the energy used is $0.997 \times 10 \times 60 = 598 \approx 600\ \text{J}$.

24 **a** Lamp C gets double the current of A and B and so it is the brightest (4 times as bright as A or B), whereas A and B are equally bright.

b In the original circuit the potential difference across A is half the emf of the battery. When C burns out, the potential difference across A is still half the emf and so there will be no change in the brightness.

c If B burns out there will be no current through A and so it will be dark. On the other hand, C will have the same potential difference across it as before (the emf) and so its brightness will stay the same.

25 **a** The voltage across the $6\ \text{k}\Omega$ is double that across the $3\ \text{k}\Omega$ resistor and the two add up to 1.5 V. Hence they must be 1.0 V and 0.5 V, respectively.

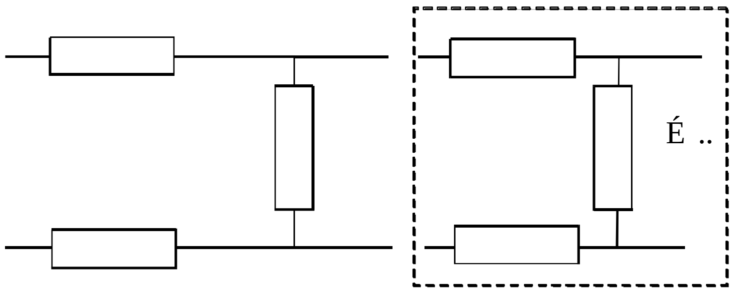
b The combination of resistor and voltmeter now has a total resistance of $1.5\ \text{k}\Omega$ and so the ratio of voltages is now 4 : 1. Hence they are 1.2 V and 0.30 V (across the voltmeter combination).

26 **a** Since $V = \varepsilon - Ir$, we will get a straight line with a positive vertical intercept and a negative slope as shown in the answers (see page 805 in *Physics for the IB Diploma*).

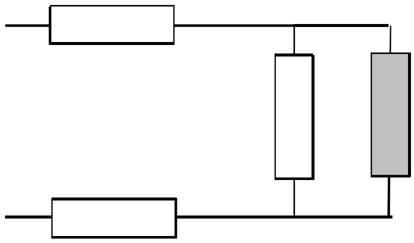
b **i** the slope is the negative of the internal resistance

ii the vertical intercept is the emf.

27 Call r the value of the resistance between A and B. Then the given arrangement can be thought to be split up as shown. Now the arrangement in the dotted line is identical to original arrangement and hence it equals r.



This means that what we have is



where the resistor in grey is that in the dotted line and so equal to r . The total resistance of this arrangement is, in stages:

$$\frac{1}{R}+\frac{1}{r}=\frac{1}{R_T}\Rightarrow R_T=\frac{rR}{r+R}$$

$$R+R+\frac{rR}{r+R}=2R+\frac{rR}{r+R}$$

But this must equal r , and so $r=2R+\frac{rR}{r+R}$

Solving, we find

$$\begin{aligned} r(r+R) &= 2R(r+R)+rR \\ r^2+rR-3rR-2R^2 &= 0 \\ r^2-2rR-2R^2 &= 0 \\ r &= \frac{2R\pm\sqrt{4R^2+8R^2}}{2} \\ &= \frac{2R\pm 2R\sqrt{3}}{2} \\ &= R(1+\sqrt{3}) \end{aligned}$$

where we take the positive root.

28 Let I be the current leaving the positive terminal of the battery. The current then comes to a junction with three wires. Because the three wires are completely symmetric with respect to each other the current will divide equally, i.e. $\frac{I}{3}$ in each. Following each of the $\frac{I}{3}$ currents we see that each reaches a junction with two wires. The situation is again symmetrical and so the current divides equally again, i.e. $\frac{I}{6}$ in each. In the next junctions the currents will begin to add up. Following the current in one closed loop containing the battery we then find:

$5.0 = \frac{I}{3}\times 1.0 + \frac{I}{6}\times 1.0 + \frac{I}{3}\times 1.0$ leading to $5.0 = \frac{5I}{6} \Rightarrow I = 6.0\text{ A}$.

29 **a** The potential difference across lamp A in both circuits is the same and therefore so is the brightness.

b If there is an internal resistance things change. The total resistance in the original circuit is:

Two lamps:

$$\frac{1}{R}+\frac{1}{R}=\frac{2}{R}$$

Total:

$$R+\frac{R}{2}=\frac{3R}{2}$$

The potential difference across the 2 lamps is therefore

$$V=\varepsilon - Ir = \varepsilon - \frac{\varepsilon}{3R/2}R = \varepsilon - \frac{2\varepsilon}{3} = \frac{\varepsilon}{3}.$$

In the new circuit:

Three lamps:

$$\frac{1}{R}+\frac{1}{R}+\frac{1}{R}=\frac{3}{R}$$

Total:

$$R+\frac{R}{3}=\frac{4R}{3}$$

The potential difference across the 3 lamps is therefore

$$V=\varepsilon - Ir = \varepsilon - \frac{\varepsilon}{4R/3}R = \varepsilon - \frac{3\varepsilon}{4} = \frac{\varepsilon}{4}$$

Hence the ratio of the brightness of lamp A in the original circuit to that in the new is $\frac{\left(\frac{\varepsilon}{3}\right)^2}{\left(\frac{\varepsilon}{4}\right)^2}=\frac{16}{9}$.

30 The potential difference across the resistance must be $12-8.0=4.0\text{ V}$ and since the current through it is 2.0 A the resistance is $R=\frac{V}{I}=\frac{4.0}{2.0}=2.0\ \Omega$.

31 Call the resistors L for left, R for right and B for bottom.

32 If B burns out and the voltmeter is ideal, there will be zero current in the circuit. Because the current is zero the presence of lamp A is irrelevant and the voltmeter reads 12 V since its ends are effectively connected to the ends of the battery (that has zero internal resistance).

33 It reads 6.0 for the same reason as in **Q32**.

34 **a** We calculate the slope to be approximately $\frac{9.3-2.4}{2.0-8.0}=-1.15$ and so the internal resistance is $r=1.15\approx 1.2\ \Omega$.

b The equation of the straight line is $V=\varepsilon-1.15I$. It goes through the point $I=8\text{ A}, V=2.4\text{ V}$ and so $2.4=\varepsilon-1.15\times 8\Rightarrow \varepsilon=11.6\approx 12\text{ V}$.

35 The potential difference across R is 1.2 V, and so the current through it is $I=\frac{1.2}{1.5}=0.80\text{ A}$. This is also the current through X.
From the graph, when the current is 0.80 A the potential difference across X is 1.6 V. Hence the emf of the battery (there is no internal resistance) is $1.2+1.6=2.8\text{ V}$.

36 When the resistors are in parallel the total resistance of the circuit is $\frac{1}{4.0}+\frac{1}{4.0}=\frac{1}{2}\Rightarrow R=2.0\ \Omega$, hence the total is $(2.0+r)\ \Omega$. Then $3.0=\frac{\varepsilon}{2.0+r}$

When in series the total resistance of the circuit is $(8.0+r)\ \Omega$ and so $1.4=\frac{\varepsilon}{8.0+r}$

a Solving these as a system we get
$3.0\times(2.0+r)=1.4\times(8.0+r)\Rightarrow r=3.25\approx 3.2\ \Omega$

and

b $\varepsilon=3\times 5.25=15.75\approx 16\text{ V}$

37 **a** The potential difference across each resistor is 1.5 V and so from the graph the current in X is 2.68 A and through Y it is 1.55 A. The total current is $4.23\approx 4.2\text{ A}$.

b If they are in series then they will take the same current, and the sum of the potential differences across each resistor will be the emf, i.e. 1.5 V. So we must use trial and error and look for horizontal lines (equal current) that intersect the two curves. We read off the voltage for each and see whether the sum is 1.5 V. This happens for approximately a current value of 1.1 A, since then the voltages are 0.5 V (X) and 1.0 V (Y).

38 Since the resistance of the PTC resistor increases, the current through it and lamp A decreases and so the brightness of A goes down. The opposite is true for the NTC resistor and so the brightness of lamp B goes up.

39 **a** It is approximately $40\ \Omega$.

b The total resistance in the circuit is $25+R_{\text{NTC}}$ and so the current is $I=\dfrac{9.0}{25+R_{\text{NTC}}}$.

The potential difference across the NTC resistor (and so the reading of the voltmeter) is then $V=IR_{\text{NTC}}=\dfrac{9.0\times R_{\text{NTC}}}{25+R_{\text{NTC}}}$.

c The total resistance of the circuit at $25\ ^\circ\text{C}$ is $40+25=65\ \Omega$ and so the current in the circuit is $I=\dfrac{9.0}{65}=0.138\ \text{A}$ and so the potential across the NTC resistor is then $0.138\times 40=5.5\ \text{V}$.

d We read the voltmeter and from $V=\dfrac{9.0\times R_{\text{NTC}}}{25+R_{\text{NTC}}}$ we determine the resistance of the NTC sensor. Then using the graph we determine the temperature.

40 **a** Since the resistors are equal, the potential differences across them are equal. They add up to 8.0 V and so each is 4.0 V.

b At normal brightness, $R=\dfrac{V}{I}=\dfrac{4.0}{0.20}=20\ \Omega$.

c The light bulb and the resistor are in parallel and have a combined resistance of $\dfrac{1}{20}+\dfrac{1}{60}=\dfrac{1}{R}\Rightarrow R=15\ \Omega$.
The potential difference across the second resistor is 4 times as large as that across the parallel combination and so the potential difference across the parallel combination (and so the lamp) is then $\dfrac{1}{5}\times 8.0=1.6\ \text{V}$.

d The current through the lamp is $I=\dfrac{1.6}{20}=0.080\ \text{A}$.

e The current is much smaller than what is required to light up the lamp and so the bulb will not light.

41 **a** The total resistance of the strain gauge–resistor combination is $50\ \Omega$ and so the voltage across the other resistor to that across the strain gauge is 2:1. Hence the voltage across the strain gauge is 2.0 V.

b The total resistance of the strain gauge–resistor combination now is $\frac{1}{110}+\frac{1}{100}=\frac{1}{R} \Rightarrow R=52.38\ \Omega$.

Hence the voltage across the strain gauge is $\frac{52.38}{52.38+100}\times 6.00=2.06\ \text{V}$.